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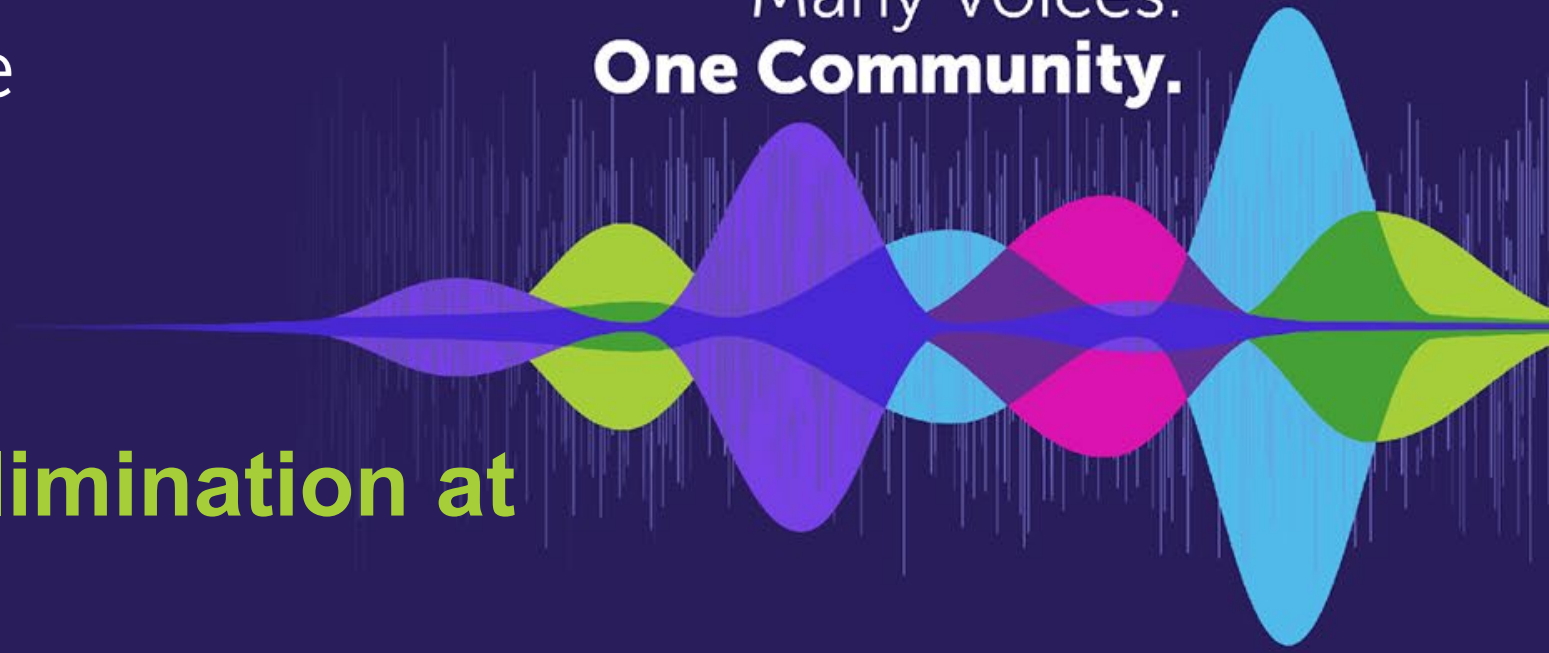

SESSION ID: CRYP-W01A

Masking Gaussian Elimination at Arbitrary Order

with Application to Multivariate- and Code-Based PQC

Quinten Norga

PhD Candidate
COSIC, KU Leuven

Co-Authors: Suparna Kundu, Uttam Kumar Ojha, Anindya Ganguly, Angshuman Karmakar, Ingrid Verbauwhede

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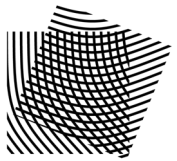
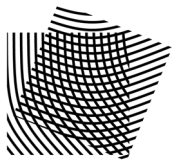
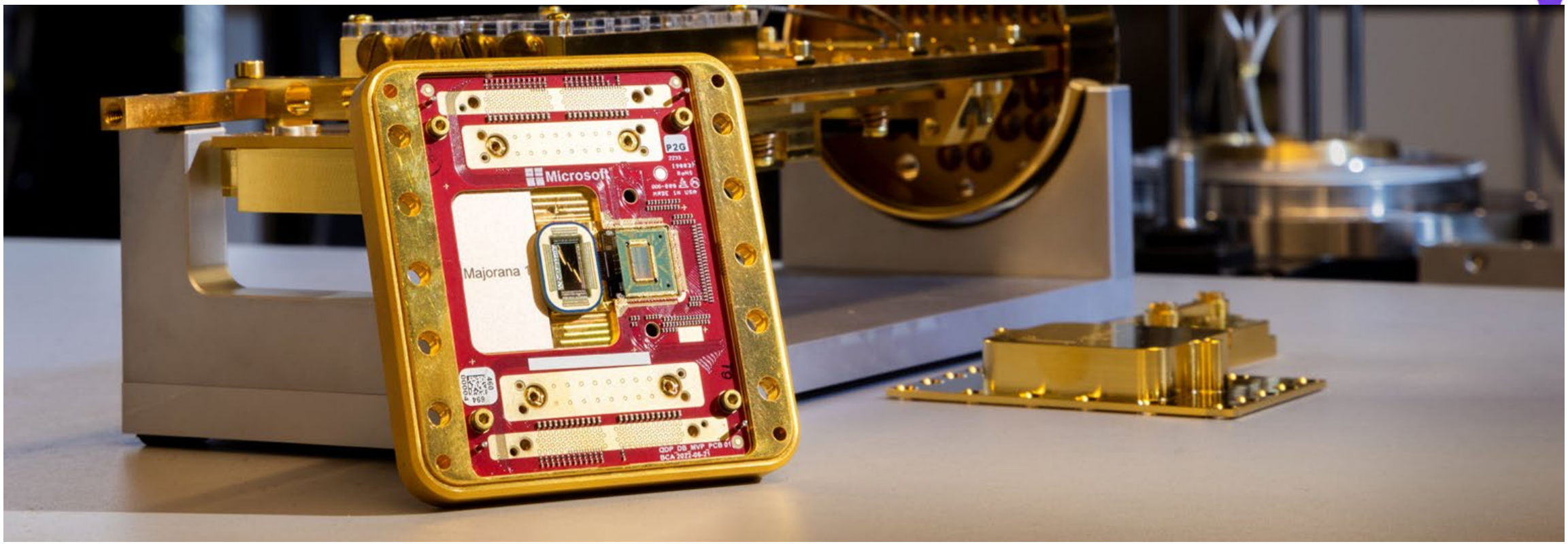


Outline

- Introduction: Multivariate- and Code-Based PQC
- Masking Gaussian Elimination w/ Back Substitution
- Implementation & Evaluation on Arm Cortex-M4



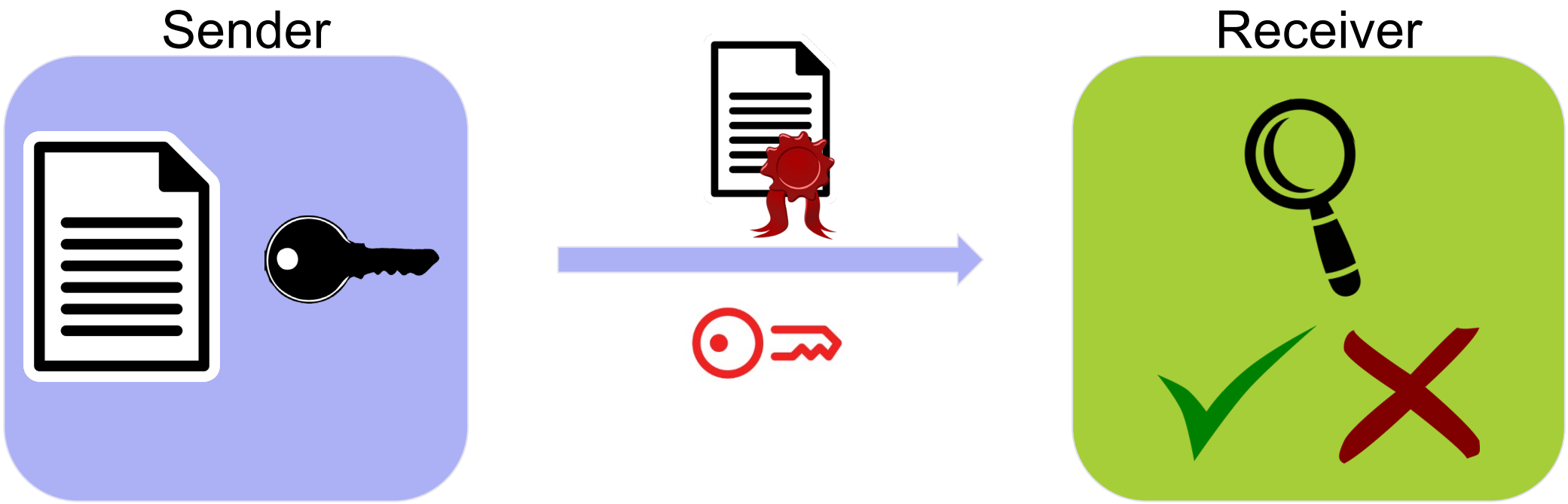
Post-Quantum Cryptography (PQC)



Majorana 1

[Microsoft]

Digital Signature Algorithm (DSA)



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PQC DSA Standardization

NIST

FIPS 204

Federal Information Processing Standards Publication

Module-Lattice-Based Digital Signature Standard

Category: Computer Security Subcategory: Cryptography

[FIPS 204]

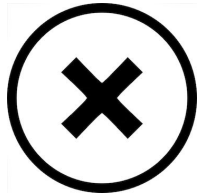
FIPS 205

Federal Information Processing Standards Publication

Stateless Hash-Based Digital Signature Standard

Category: Computer Security Subcategory: Cryptography

[FIPS 205]



and many more…

[NIST]

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Multivariate- and Code-Based DSA

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

System of Linear Equations

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Linear Algebra 101: Gaussian Elimination

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} & b_1 \\ a_{21} & a_{22} & \dots & a_{2m} & b_2 \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mm} & b_m \end{bmatrix}$$

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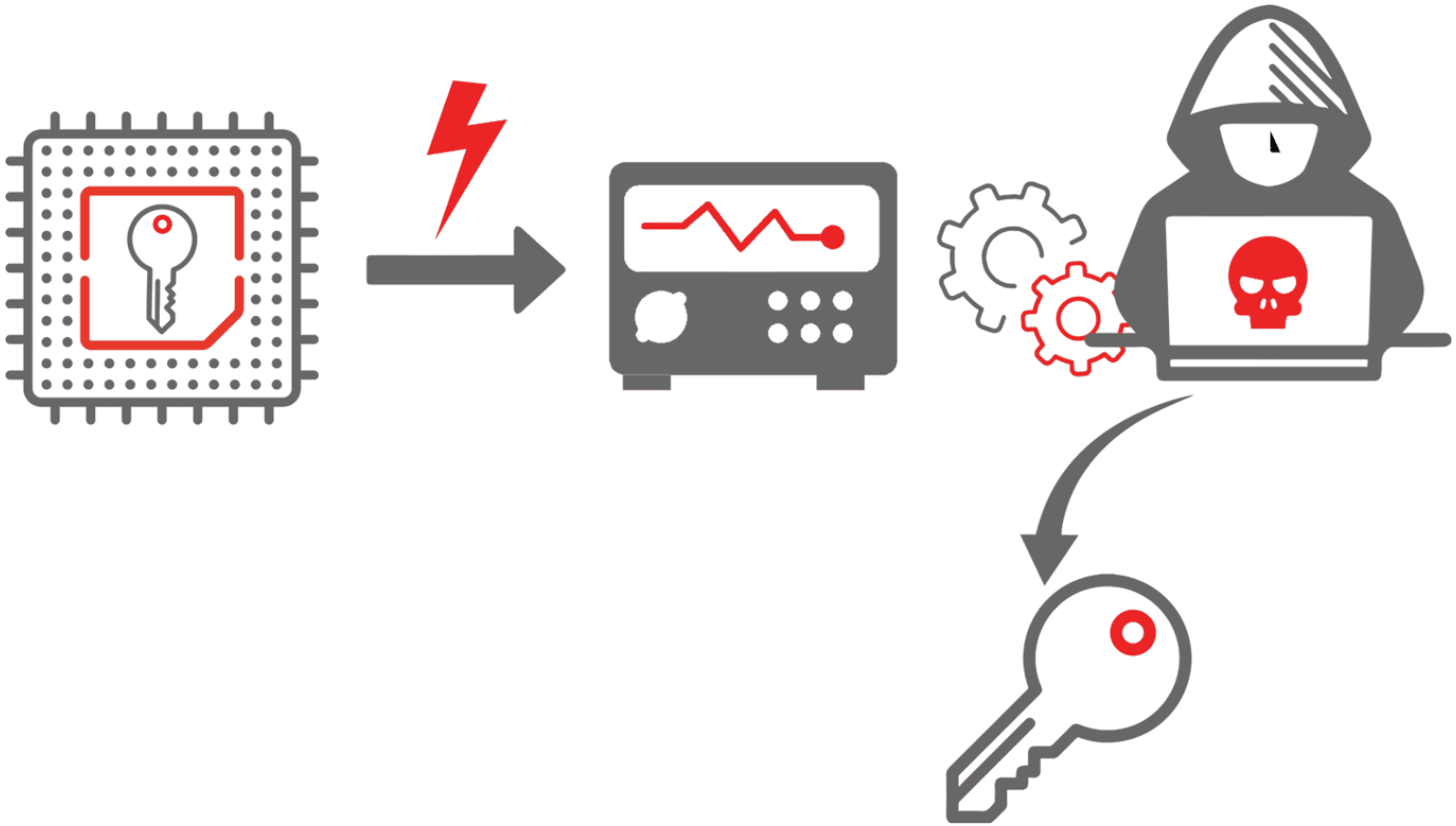
Row-Echelon Form

$$\begin{bmatrix} 1 & a'_{12} & \dots & a'_{1m} & b'_1 \\ 0 & 1 & \dots & a'_{2m} & b'_2 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & b'_m \end{bmatrix}$$

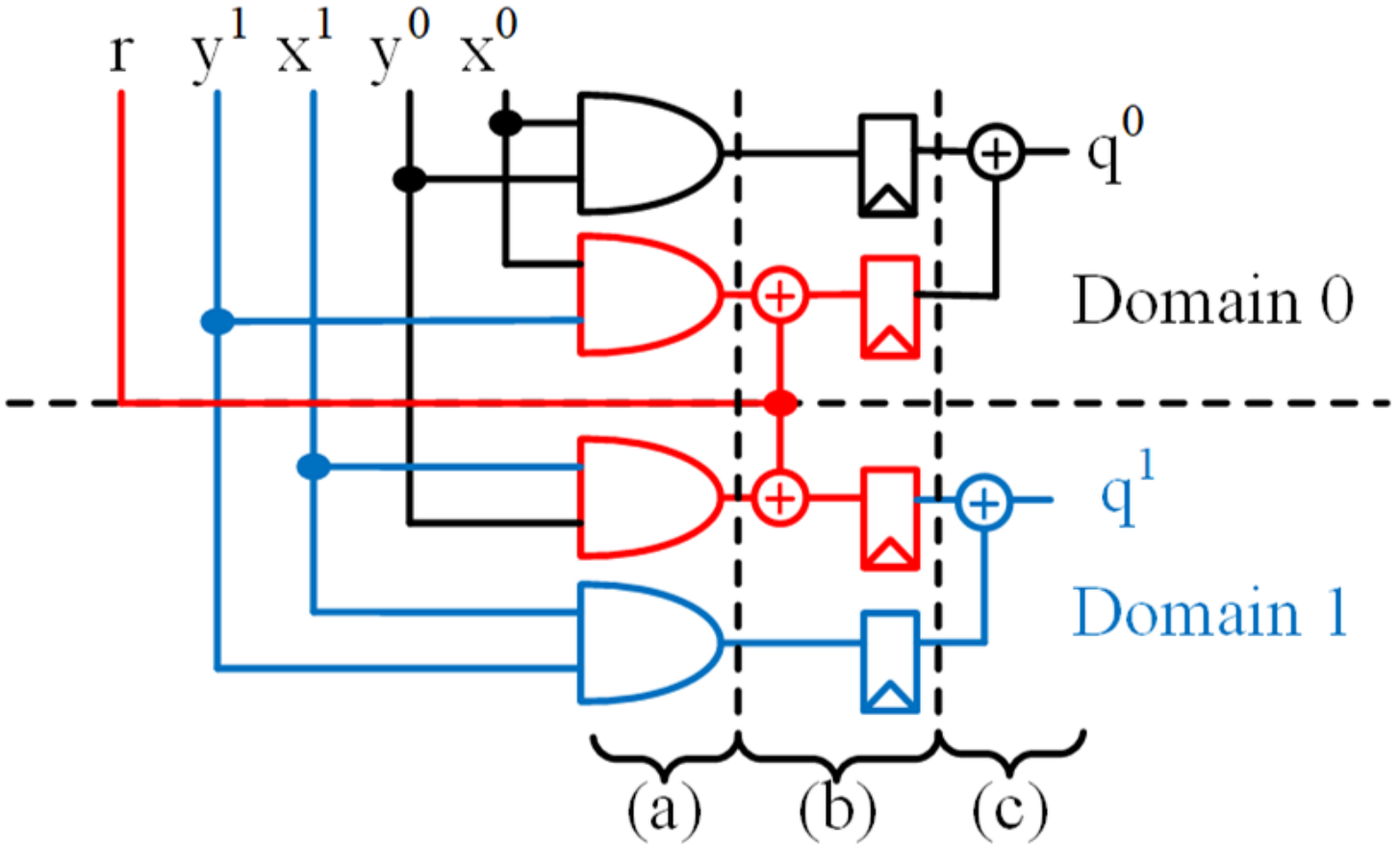
Row-Operations: Swap, Multiply, Add

Side-Channel Attacks (SCAs)



[Secure-IC]

Masking



[Gross et al.]

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Masked System of Linear Equations

$$\begin{bmatrix} (a^1_{11}, a^2_{11}) & (a^1_{12}, a^2_{12}) & \dots & (a^1_{12}, a^2_{12}) & (b^1_1, b^2_1) \\ (a^1_{21}, a^2_{21}) & (a^1_{22}, a^2_{22}) & \dots & (a^1_{2m}, a^2_{2m}) & (b^1_2, b^2_2) \\ \dots & \dots & \dots & \dots & \dots \\ (a^1_{m1}, a^2_{m1}) & (a^1_{m2}, a^2_{m2}) & \dots & (a^1_{mm}, a^2_{mm}) & (b^1_m, b^2_m) \end{bmatrix}$$

$$a^1_{xy} * a^2_{xy} = a_{xy}$$

Multiplicative Sharing

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Masking Gaussian Elimination

Step 1: Make Pivot Non-Zero

$$\begin{bmatrix} (a^1_{11}, a^2_{11}) & (a^1_{12}, a^2_{12}) & \dots & (a^1_{12}, a^2_{12}) & (b^1_1, b^2_1) \\ (a^1_{21}, a^2_{21}) & (a^1_{22}, a^2_{22}) & \dots & (a^1_{2m}, a^2_{2m}) & (b^1_2, b^2_2) \\ \dots & \dots & \dots & \dots & \dots \\ (a^1_{m1}, a^2_{m1}) & (a^1_{m2}, a^2_{m2}) & \dots & (a^1_{mm}, a^2_{mm}) & (b^1_m, b^2_m) \end{bmatrix}$$

$$a^1_{11} + a^2_{11} = a_{11} \stackrel{?}{=} 0$$

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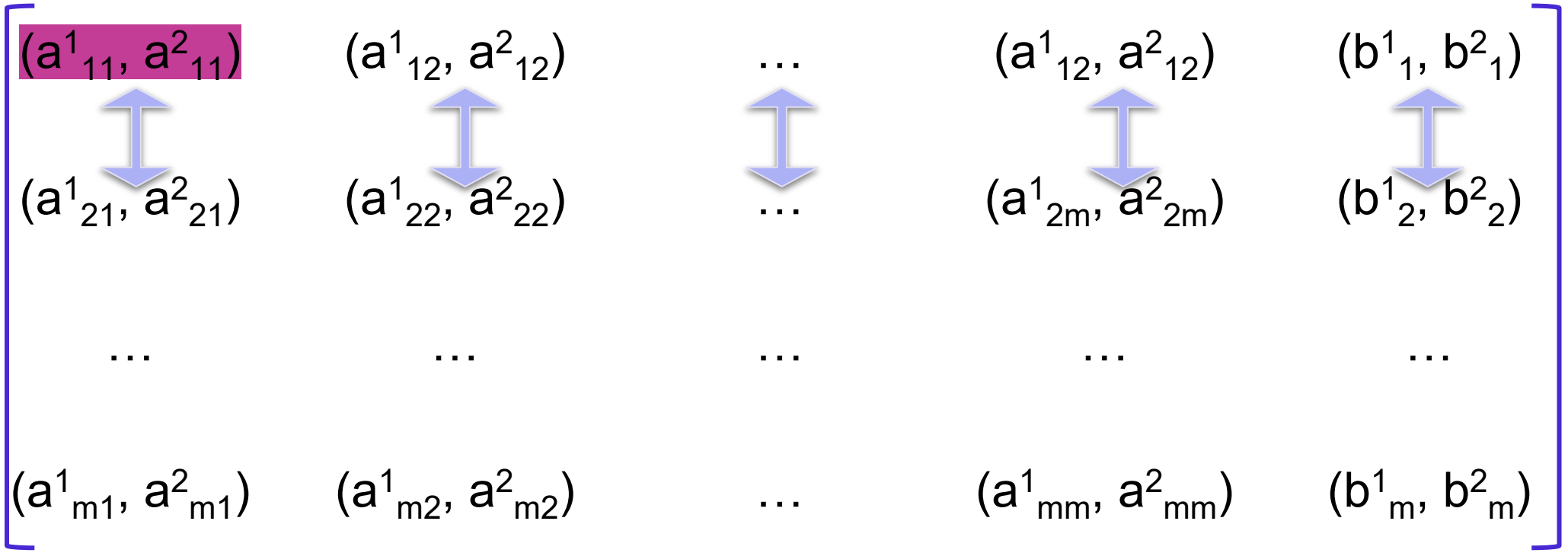
SecNonzero [CC24]

$$a^1_{11} + a^2_{11} = a_{11} \stackrel{?}{=} 0$$

- Returns masked bool (1/0)
- Unmasking *leaks* information

Step 1: Make Pivot Non-Zero



$a^1_{11} + a^2_{11} = a_{11} \neq 0$

Not Constant Time!

SecCondAdd

- Conditional Addition
  - Add rows to Pivot row…
  - or add 0 to Pivot row

$$a^1_{11} + a^2_{11} = a_{11} = 0$$

$$a^1_{11} + a^2_{11} = a_{11} \neq 0$$

  - Details in paper

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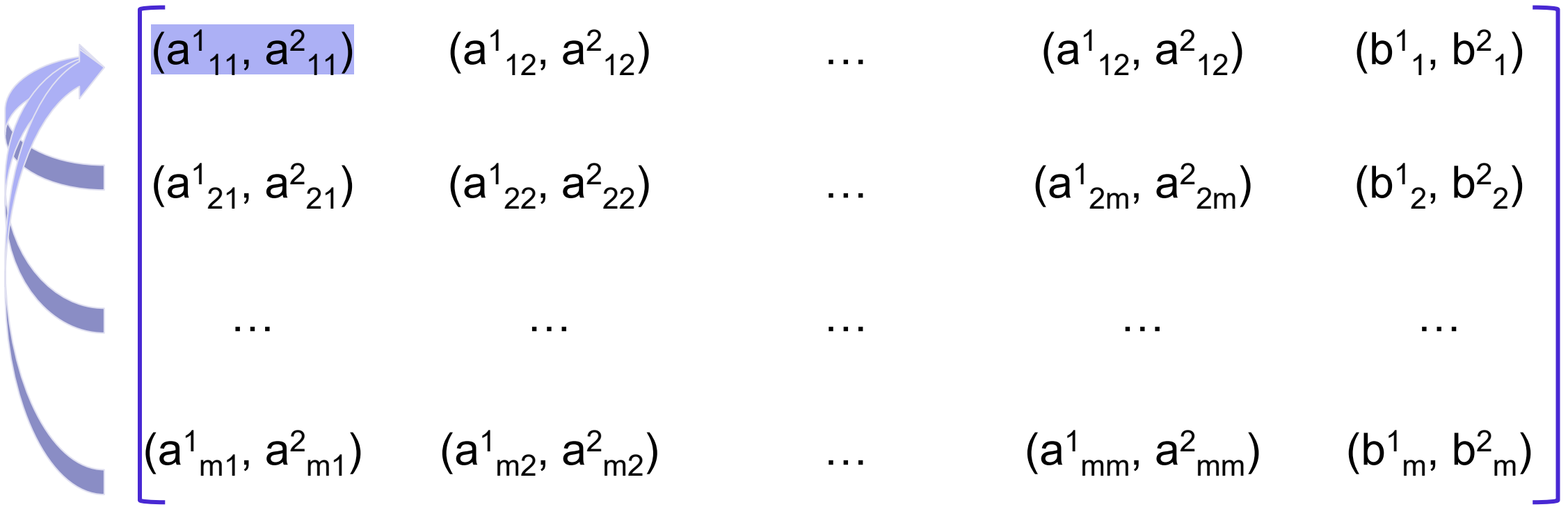
Step 1: Make Pivot Non-Zero



SecCondAdd (CT)

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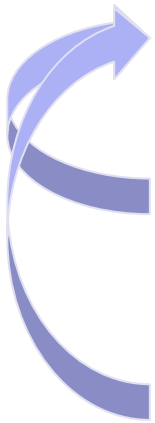
Step 1: Make Pivot Non-Zero



$$\begin{bmatrix} (a^1_{11}, a^2_{11}) & (a^1_{12}, a^2_{12}) & \dots & (a^1_{12}, a^2_{12}) & (b^1_1, b^2_1) \\ (a^1_{21}, a^2_{21}) & (a^1_{22}, a^2_{22}) & \dots & (a^1_{2m}, a^2_{2m}) & (b^1_2, b^2_2) \\ \dots & \dots & \dots & \dots & \dots \\ (a^1_{m1}, a^2_{m1}) & (a^1_{m2}, a^2_{m2}) & \dots & (a^1_{mm}, a^2_{mm}) & (b^1_m, b^2_m) \end{bmatrix}$$

Early Stop (details in paper)

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Step 2: Check Pivot

$$\begin{bmatrix} (a^1_{11}, a^2_{11}) & (a^1_{12}, a^2_{12}) & \dots & (a^1_{12}, a^2_{12}) & (b^1_1, b^2_1) \\ (a^1_{21}, a^2_{21}) & (a^1_{22}, a^2_{22}) & \dots & (a^1_{2m}, a^2_{2m}) & (b^1_2, b^2_2) \\ \dots & \dots & \dots & \dots & \dots \\ (a^1_{m1}, a^2_{m1}) & (a^1_{m2}, a^2_{m2}) & \dots & (a^1_{mm}, a^2_{mm}) & (b^1_m, b^2_m) \end{bmatrix}$$

SecNonzero

SecNonzero [CC24] + Unmask

$$a^1_{11} + a^2_{11} = a_{11} \stackrel{?}{=} 0$$

- Leak if pivot is (non-)zero
- No unique solution → OK!

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Step 2: Check Pivot

$$\begin{bmatrix} a_{11}=0 & (a^1_{12}, a^2_{12}) & \dots & (a^1_{12}, a^2_{12}) & (b^1_1, b^2_1) \\ (a^1_{21}, a^2_{21}) & (a^1_{22}, a^2_{22}) & \dots & (a^1_{2m}, a^2_{2m}) & (b^1_2, b^2_2) \\ \dots & \dots & \dots & \dots & \dots \\ (a^1_{m1}, a^2_{m1}) & (a^1_{m2}, a^2_{m2}) & \dots & (a^1_{mm}, a^2_{mm}) & (b^1_m, b^2_m) \end{bmatrix}$$

Start over!

Step 3: Make Pivot 1

$$\begin{bmatrix} (a^1_{11}, a^2_{11}) & (a^1_{12}, a^2_{12}) & \dots & (a^1_{12}, a^2_{12}) & (b^1_1, b^2_1) \\ (a^1_{21}, a^2_{21}) & (a^1_{22}, a^2_{22}) & \dots & (a^1_{2m}, a^2_{2m}) & (b^1_2, b^2_2) \\ \dots & \dots & \dots & \dots & \dots \\ (a^1_{m1}, a^2_{m1}) & (a^1_{m2}, a^2_{m2}) & \dots & (a^1_{mm}, a^2_{mm}) & (b^1_m, b^2_m) \end{bmatrix}$$

$$a_{11} * a_{11}^{-1} = 1$$

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B2Minv

- Multiplication prefers multiplicative masking
  - B2M conversion [GenProQui11]

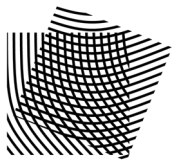
$$(a^1_{11} + a^2_{11}) = (a'^1_{11} * a'^2_{11})$$

SecScalarMult

- Strictly* linear operations

$$(a^1_{11} + a^2_{11}) * a'^1_{11} * a'^2_{11}$$

- Total Cost:
  - 1 B2M conversion
  - NO M2B conversion



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Step 3: Make Pivot 1

$$\begin{bmatrix} (a^1_{11}, a^2_{11}) & (a^1_{12}, a^2_{12}) & \dots & (a^1_{12}, a^2_{12}) & (b^1_1, b^2_1) \\ (a^1_{21}, a^2_{21}) & (a^1_{22}, a^2_{22}) & \dots & (a^1_{2m}, a^2_{2m}) & (b^1_2, b^2_2) \\ \dots & \dots & \dots & \dots & \dots \\ (a^1_{m1}, a^2_{m1}) & (a^1_{m2}, a^2_{m2}) & \dots & (a^1_{mm}, a^2_{mm}) & (b^1_m, b^2_m) \end{bmatrix}$$

$$(a^1_{11} + a^2_{11}) = 1$$

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Step 3: Make Pivot 1

$$\begin{bmatrix} (a^1_{11}, a^2_{11}) & (a^1_{12}, a^2_{12}) & \dots & (a^1_{12}, a^2_{12}) & (b^1_1, b^2_1) \\ (a^1_{21}, a^2_{21}) & (a^1_{22}, a^2_{22}) & \dots & (a^1_{2m}, a^2_{2m}) & (b^1_2, b^2_2) \\ \dots & \dots & \dots & \dots & \dots \\ (a^1_{m1}, a^2_{m1}) & (a^1_{m2}, a^2_{m2}) & \dots & (a^1_{mm}, a^2_{mm}) & (b^1_m, b^2_m) \end{bmatrix}$$

SecScalarMult on Row 1

Step 4: Make Lower Elements 0

$$\begin{bmatrix} (a^1_{11}, a^2_{11}) & (a^1_{12}, a^2_{12}) & \dots & (a^1_{12}, a^2_{12}) & (b^1_1, b^2_1) \\ (a^1_{21}, a^2_{21}) & (a^1_{22}, a^2_{22}) & \dots & (a^1_{2m}, a^2_{2m}) & (b^1_2, b^2_2) \\ \dots & \dots & \dots & \dots & \dots \\ (a^1_{m1}, a^2_{m1}) & (a^1_{m2}, a^2_{m2}) & \dots & (a^1_{mm}, a^2_{mm}) & (b^1_m, b^2_m) \end{bmatrix}$$

Subtract first-row from rows below: SecMultSub

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Step 4: Make Lower Elements 0

$$\begin{bmatrix} (a^1_{11}, a^2_{11}) & (a^1_{12}, a^2_{12}) & \dots & (a^1_{12}, a^2_{12}) & (b^1_1, b^2_1) \\ (a^1_{21}, a^2_{21}) & (a^1_{22}, a^2_{22}) & \dots & (a^1_{2m}, a^2_{2m}) & (b^1_2, b^2_2) \\ \dots & \dots & \dots & \dots & \dots \\ (a^1_{m1}, a^2_{m1}) & (a^1_{m2}, a^2_{m2}) & \dots & (a^1_{mm}, a^2_{mm}) & (b^1_m, b^2_m) \end{bmatrix}$$

Subtract first-row from rows below: SecMultSub

New Pivot: Repeat Step 1-4

$$\begin{bmatrix} (a^1_{11}, a^2_{11}) & (a^1_{12}, a^2_{12}) & \dots & (a^1_{12}, a^2_{12}) & (b^1_1, b^2_1) \\ (a^1_{21}, a^2_{21}) & (a^1_{22}, a^2_{22}) & \dots & (a^1_{2m}, a^2_{2m}) & (b^1_2, b^2_2) \\ \dots & \dots & \dots & \dots & \dots \\ (a^1_{m1}, a^2_{m1}) & (a^1_{m2}, a^2_{m2}) & \dots & (a^1_{mm}, a^2_{mm}) & (b^1_m, b^2_m) \end{bmatrix}$$

$$a^1_{22} + a^2_{22} = a_{22} \stackrel{?}{=} 0$$

Resulting Matrix

$$\begin{bmatrix} (a^1_{11}, a^2_{11}) & (a^1_{12}, a^2_{12}) & \dots & (a^1_{12}, a^2_{12}) & (b^1_1, b^2_1) \\ (a^1_{21}, a^2_{21}) & (a^1_{22}, a^2_{22}) & \dots & (a^1_{2m}, a^2_{2m}) & (b^1_2, b^2_2) \\ \dots & \dots & \dots & \dots & \dots \\ (a^1_{m1}, a^2_{m1}) & (a^1_{m2}, a^2_{m2}) & \dots & (a^1_{mm}, a^2_{mm}) & (b^1_m, b^2_m) \end{bmatrix}$$

Final step: back-substitution

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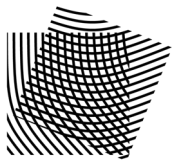
SecBackSub

- See paper for details

- Reduce cost through early unmasking



Application to PQC DS Schemes

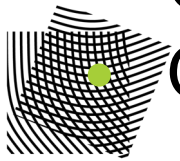
- Cost determined by
  - **m** (matrix dimension)
  - **q** (modulus)
  - **n** (# shares)

- NIST Additional DS Round 2 & Korean PQC Candidates
  - UOV, MAYO, SNOVA, QR-UOV, MQ-Sign

UOV Implementation on Arm Cortex-M4

Scheme	Operation	Umasked	Masked		
			1st-order	2nd-order	3rd-order
UOV-I (m=44 q=256)	GE (Total)	1034	**15649 (15.1×)**	**39587 (38.3×)**	**69186 (66.9×)**
	SecRowEch	999	15575 (15.6×)	39469 (39.5×)	69021 (69.1×)
	Step 1	80	1998 (25×)	5642 (70.5×)	9901 (123.8×)
	Step 2		39	108	193
	Step 3		598	1335	2426
	Step 4		12938	32383	56498
	SecBackSub	32	74 (2.3×)	118 (3.7×)	164 (5.1×)
UOV-III	GE (Total)	4117	**62680 (15.2×)**	**157901 (38.4×)**	**275768 (67×)**
UOV-V	GE (Total)	9336	**143361 (15.4×)**	**360590 (38.6×)**	**1025942 (109.9×)**

- STM32 Discovery
- C code (arm-none-eabi-gcc)
- On-the-fly TRNG



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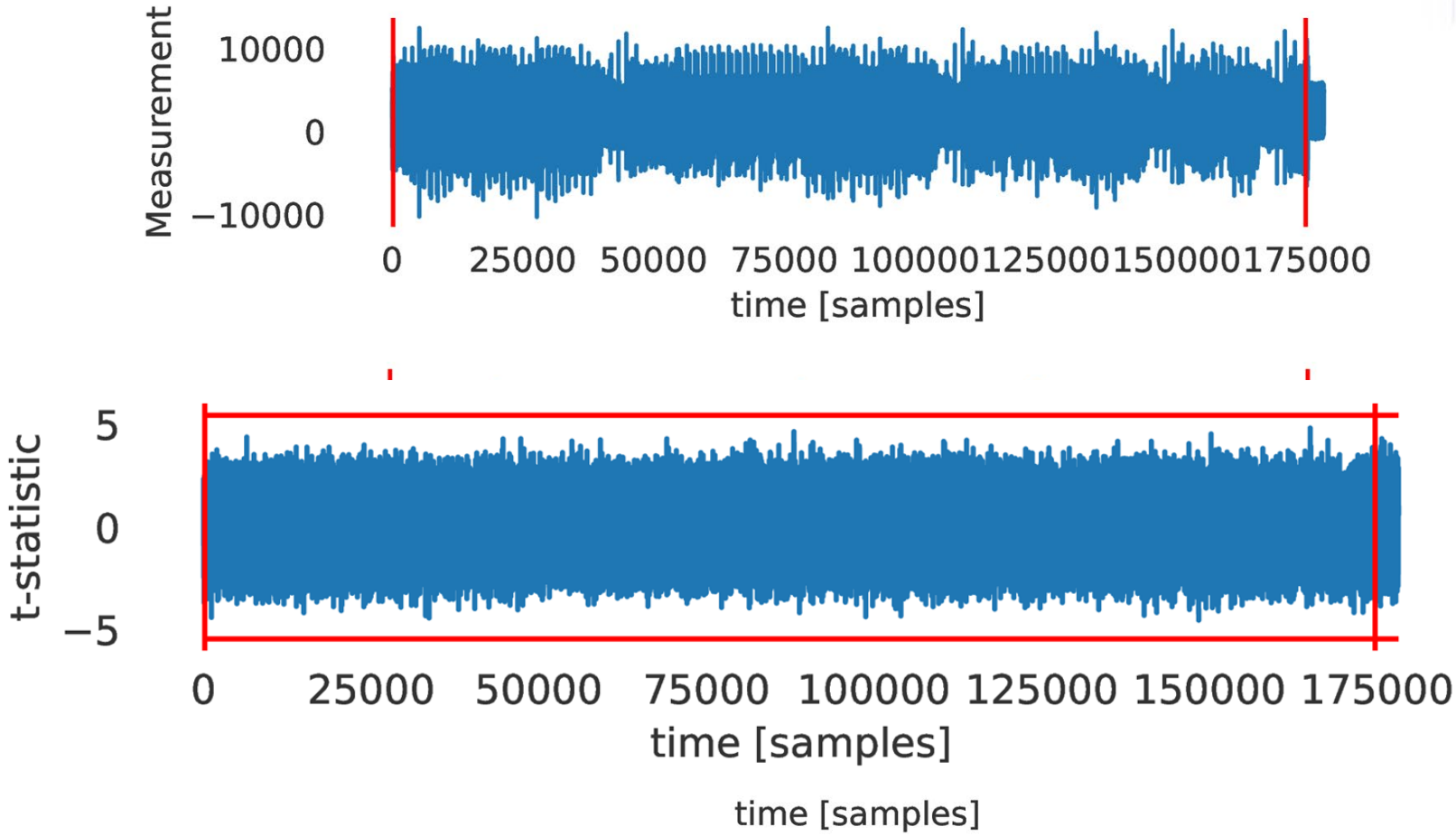
Security Evaluation in COSIC Lab



Countermeasure OFF

Countermeasure ON

Test Vector Leakage Assessment (TVLA)

Mitigation of micro-architectural leakages

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Key Takeaways

How to Apply our results?

Conclusion

- **Real-World** requires **Secure & Efficient** implementations
  - Design of PQC schemes
  - Implementation of PQC schemes

- PQC transition
  - Crypto inventory
  - Agility: many standards!

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Thank You!

Questions?

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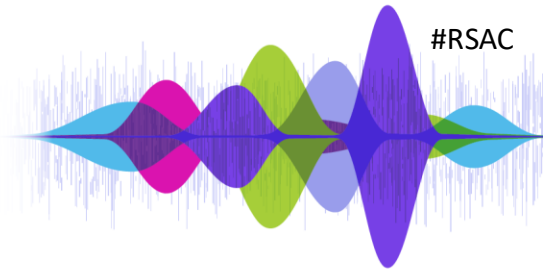
SESSION ID: CRYP-W01B

Hardware and Software Implementations: Post-Quantum Online/Offline Signatures

James Howe

Staff Research Scientist
SandboxAQ
https://www.linkedin.com/in/jameshowe1729/



Outline

- Introduction
  - A refresher on Post-Quantum Cryptography
  - What is the Online/Offline paradigm
  - Why could this intersection work?
- Our Contributions
- Results

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Introduction

Why post-quantum cryptography matters

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Why Post-Quantum Matters

The Quantum Threat

Practical quantum computers are coming

RSA & ECC vulnerable via Shor's algorithm

NIST's PQC Efforts

Standardizing quantum-secure cryptosystems.

FIPS standards: ML-KEM, ML-DSA (Dilithium), SLH-DSA, & Falcon.

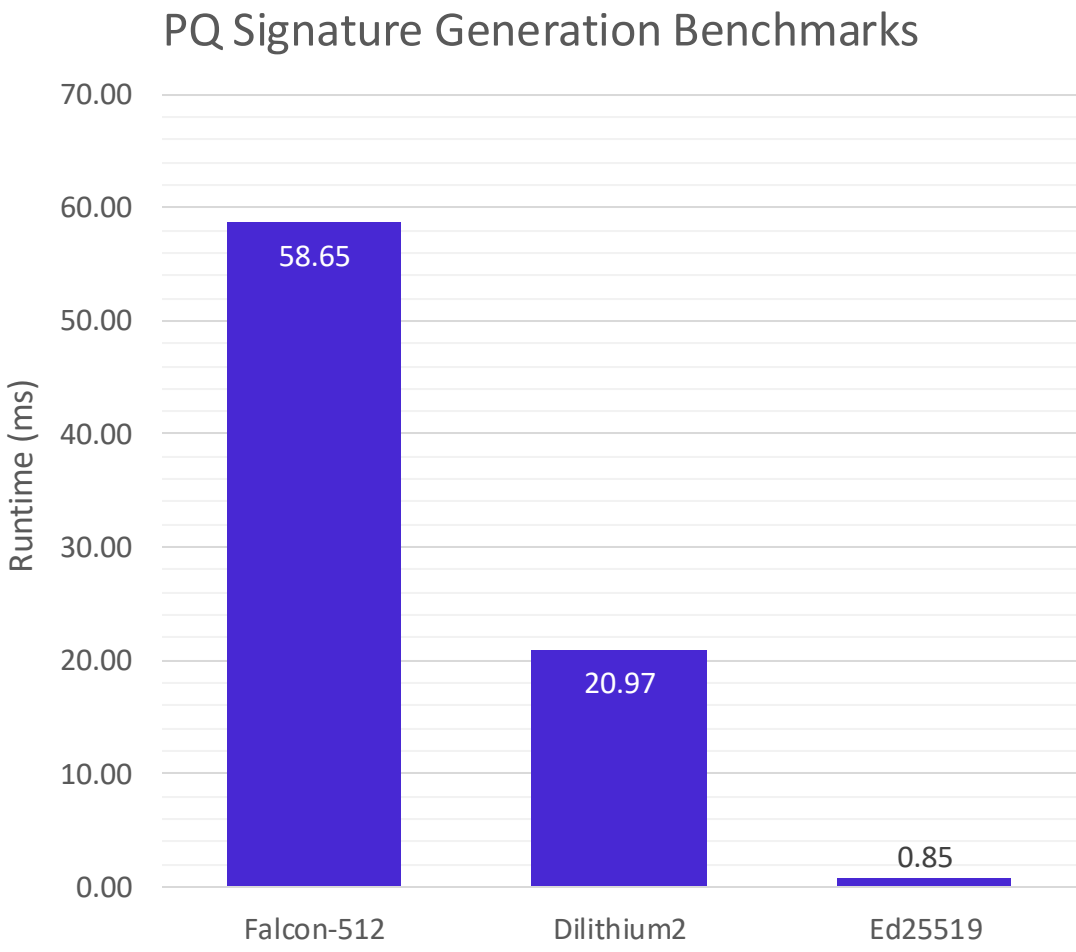
PQC Signature Issues

High computational costs → Slower than classical signatures.

Impacts smart cards, IoT, embedded devices.

PQ Signature Embedded Performance

- Why This Matters
  - Ed25519 is fast (used today in SSH, TLS, etc.).
  - Falcon & Dilithium are much slower due to heavy math operations.
- **Impact**: Real-time applications struggle with PQC signatures.
- **Solution**: Online/Offline signatures
  - Precompute heavy operations “offline” to make signing faster.



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Introduction

What are online/offline signatures?

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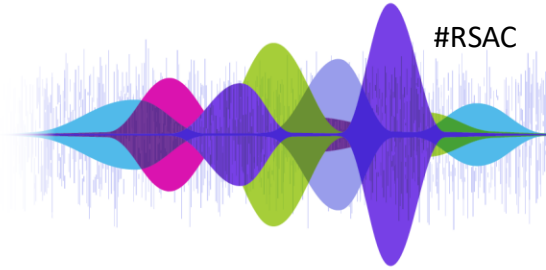
What are online/offline signatures?

- Introduced by Even, Goldreich, & Micali (1989, 1996)
- Allows efficient signing on devices with limited computational power.
  - 💳 Smart Cards – auth for ID cards, secure access, etc.
  - 🏪 POS (Point-of-Sale) Payment Systems – NFC & RFID-based contactless payments.
  - 🔐 Secure Transactions – Used in low-power devices where fast signing is required.



On-Line/Off-Line Digital Signatures*

Shimon Even
Computer Science Department, Technion—Israel Institute of Technology, Haifa 32000, Israel
even@cs.technion.ac.il

Oded Goldreich
Department of Applied Mathematics and Computer Science, Weizmann Institute of Science, Rehovot, Israel
oded@wisdom.weizmann.ac.il

Silvio Micali
Laboratory for Computer Science, Massachusetts Institute of Technology, 545 Technology Square, Cambridge, MA 02139, U.S.A.
silvio@theory.lcs.mit.edu

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Received 19 August 1992 and revised 21 December 1994

Abstract. A new type of signature scheme is proposed. It consists of two phases. The first phase is performed off-line, before the message to be signed is even known. The second phase is performed on-line, once the message to be signed is known, and is supposed to be very fast. A method for constructing such on-line/off-line signature schemes is presented. The method uses one-time signature schemes, which are very fast, for the on-line signing. An ordinary signature scheme is used for the off-line stage.

In a practical implementation of our scheme, we use a variant of Rabin's signature scheme (based on factoring) and DES. In the on-line phase all we use is a moderate amount of DES computation and a single modular multiplication. We stress that the costly modular exponentiation operation is performed off-line. This implementation is ideally suited for electronic wallets or smart cards.

Key words. Digital signatures, Integer factorization, RSA, DES, One-time signature schemes, Error-correcting codes, Chosen message attack.

What are online/offline signatures?

- Splits signing into two phases:
  - Offline Phase 🛠️ → Precompute expensive operations before signing (e.g., modular exponentiation)
  - Online Phase ⚡ → Fast signing when needed using precomputed data

On-Line/Off-Line Digital Signatures*

Shimon Even
Computer Science Department, Technion—Israel Institute of Technology, Haifa 32000, Israel
even@cs.technion.ac.il

Oded Goldreich
Department of Applied Mathematics and Computer Science, Weizmann Institute of Science, Rehovot, Israel
oded@wisdom.weizmann.ac.il

Silvio Micali
Laboratory for Computer Science, Massachusetts Institute of Technology, 545 Technology Square, Cambridge, MA 02139, U.S.A.
silvio@theory.lcs.mit.edu

Communicated by Gilles Brassard

Received 19 August 1992 and revised 21 December 1994

Abstract. A new type of signature scheme is proposed. It consists of two phases. The first phase is performed off-line, before the message to be signed is even known. The second phase is performed on-line, once the message to be signed is known, and is supposed to be very fast. A method for constructing such on-line/off-line signature schemes is presented. The method uses one-time signature schemes, which are very fast, for the on-line signing. An ordinary signature scheme is used for the off-line stage.

In a practical implementation of our scheme, we use a variant of Rabin's signature scheme (based on factoring) and DES. In the on-line phase all we use is a moderate amount of DES computation and a single modular multiplication. We stress that the costly modular exponentiation operation is performed off-line. This implementation is ideally suited for electronic wallets or smart cards.

Key words. Digital signatures, Integer factorization, RSA, DES, One-time signature schemes, Error-correcting codes, Chosen message attack.

What are online/offline signatures?

- Offline precomputation phase 🛠️
  - Using a regular signature scheme, generates sign/verify long-term keys
  - Generates one-time sign/verify keys
  - Signs the one-time verify key with long term key
- Online Phase ⚡
  - Quickly verifies the one-time key
  - Signs the message/challenge with the one-time key
  - Quickly verifies the one-time signature created
- ⚠️ We are verifying twice, only works if 2*verifications are much faster than one regular signing

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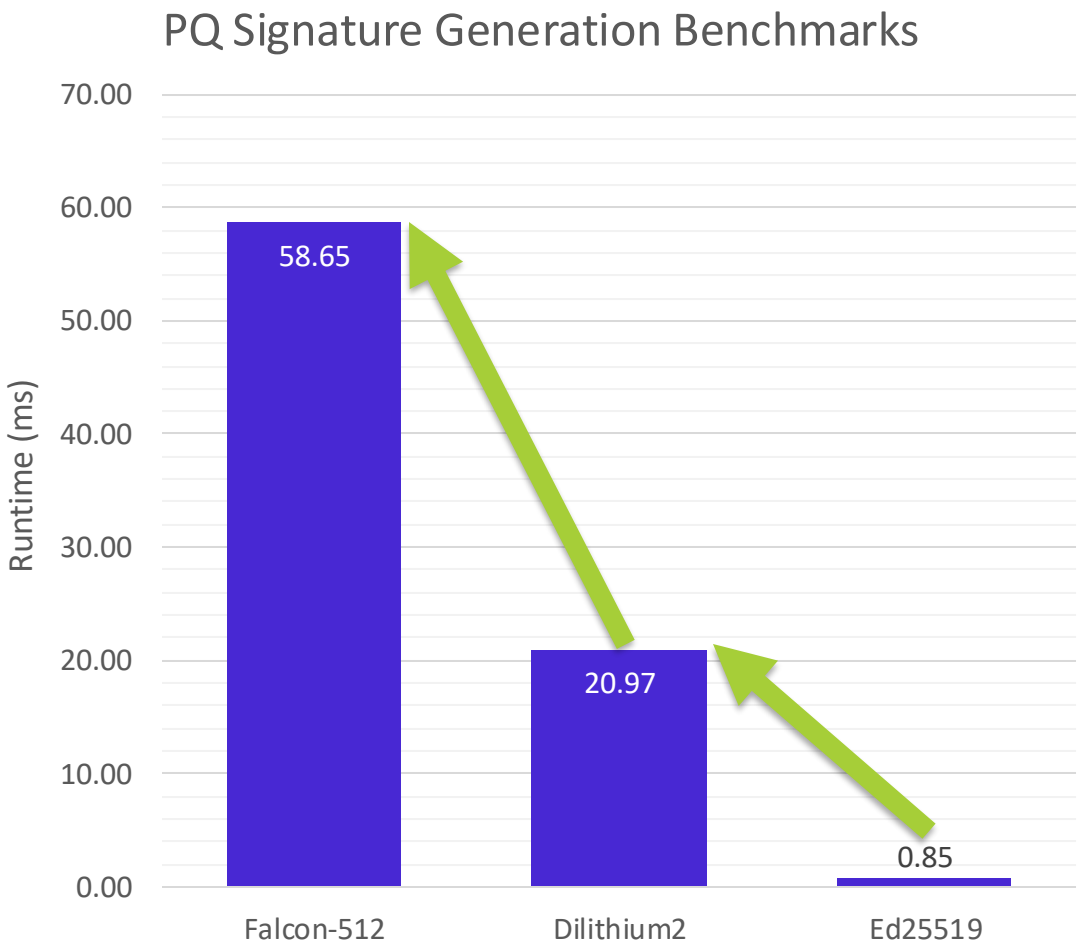
Post-Quantum Online/Offline Signatures

Why Falcon?

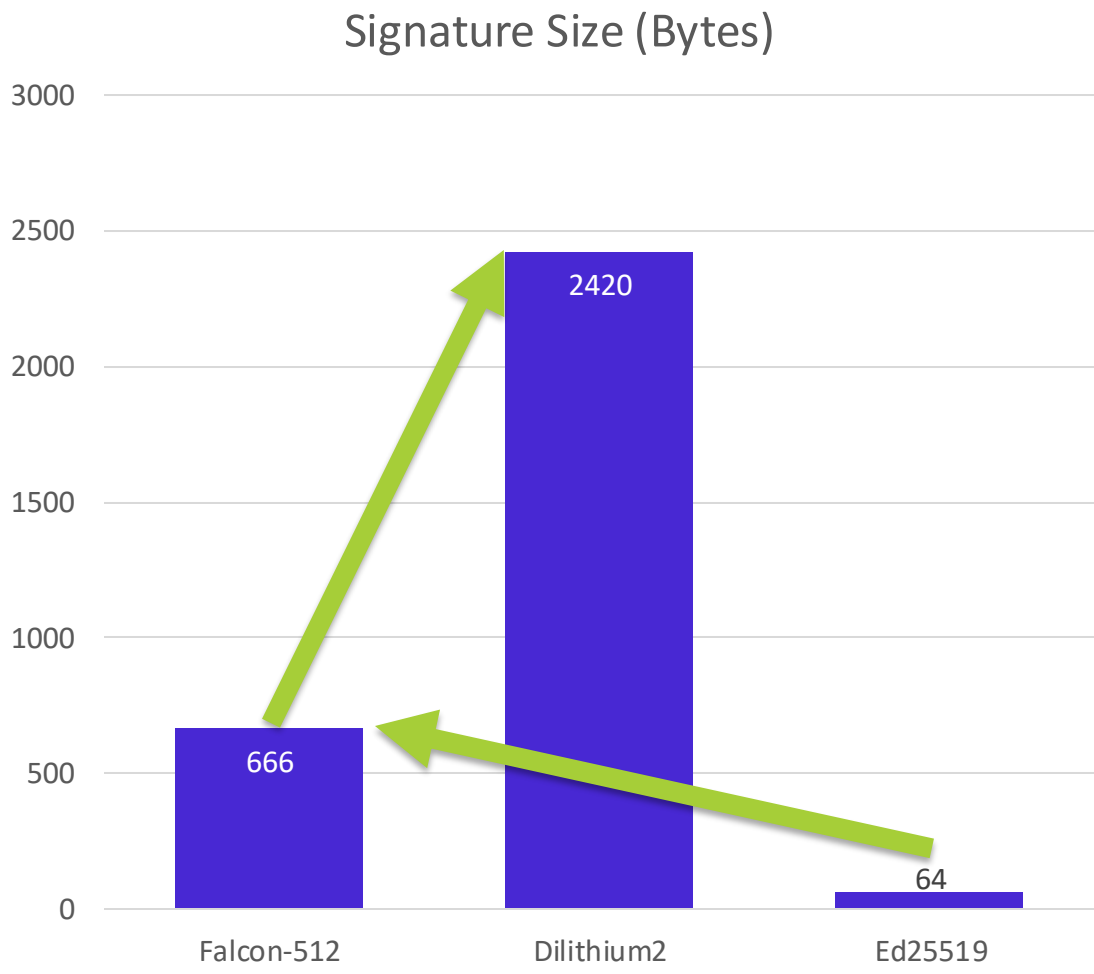
Why Falcon?

- At this point you may be wondering why not Dilithium?
  - Dilithium is much faster than Falcon!
  - There’s an order of magnitude between them!
  - Surely, it's a better choice for online/offline?



Why Falcon?

- Falcon more suited to small devices:
  - Compact Signatures 📏 – Falcon is 3.5x smaller
  - Fast Verification ⚡ – Falcon verifies 3x faster
  - Falcon is also a NIST PQC standard
- Our goal:
  - Apply Online/Offline to Falcon's slower signing times!
  - Retain Falcon's advantages
  - Make POS etc. PQ and practical!



Falcon: Under the Hood

Why is Falcon so expensive to sign?

- Many *FFT* conversions of the secret key
  - Costs more than total Dilithium2 signing
  - And this happens twice!
- *ffSampling* – trapdoor Gaussian sampler
  - It's recursive, uses a lot of randomness, and is computationally expensive
- Falcon requires *floating-point* operations
  - And on small embedded devices with no FPU means costly emulation

$\mathsf{Sign}(\mathsf{sk}, \mu, \lfloor \beta^2 \rfloor)$

$\mathbf{r} \leftarrow\!\!\$\ \{0,1\}^{320}$

$c \leftarrow \mathsf{HashToPoint}(\mathbf{r} \| \mu, q, n)$

$\mathbf{t} \leftarrow (-\frac{1}{q}\mathsf{FFT}(c) \odot \mathsf{FFT}(F), -\frac{1}{q}\mathsf{FFT}(c) \odot \mathsf{FFT}(f))$

do

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    $\mathbf{z} \leftarrow \mathsf{ffSampling}_n(\mathbf{t}, \mathbf{T})$;

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  while $\|\mathbf{s}\|^2 > \lfloor \beta^2 \rfloor$

  $(s_0, s_1) \leftarrow \mathsf{invFFT}(\mathbf{s})$

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while $s = \perp$

return $\sigma := (\mathbf{r}, s)$

Lazy Falcon

Our Contributions

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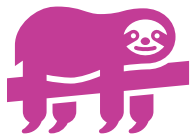
Offline

Lazify



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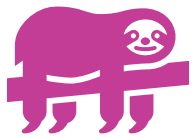

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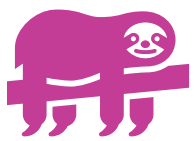

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Post-Quantum Online/Offline Signatures

Lazy Falcon “Few Times” Signatures

Many Voices.
One Community.

Lazy Falcon

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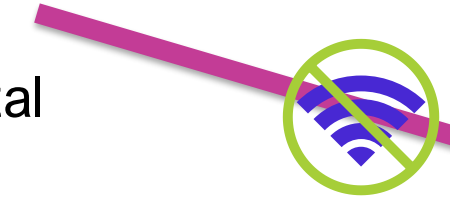

PreSign(sk)

$\mathbf{B} \leftarrow [g, -f; G, -F]$

$\hat{\mathbf{B}} \leftarrow \mathsf{FFT}(\mathbf{B})$

$u_0, u_1 \leftarrow \mathcal{D}_\sigma^2$

$c_{\text{pre}} \leftarrow \mathsf{ComputeTarget}(h, u_0, u_1)$

return $(\rho := u_0, u_1, c_{\text{pre}})$

ComputeTarget(h, u_0, u_1)

$\hat{u_1} \leftarrow \mathsf{NTT}(u_1)$

$\hat{t} \leftarrow h \odot \hat{u_1}$

$t \leftarrow \mathsf{invNTT}(\hat{t}) + u_0$

return t

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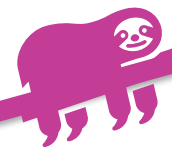

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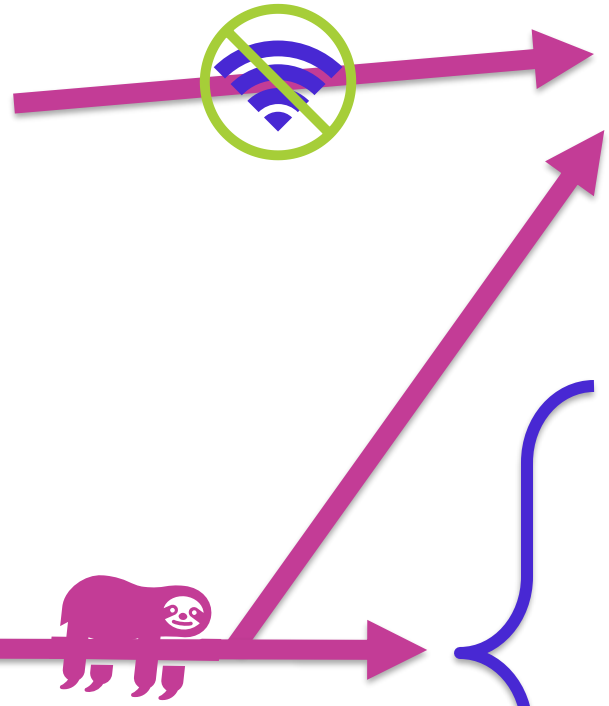

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$c' \leftarrow \mathsf{HashToPoint}(\mathbf{r} \| \mu, q, n)$

$c \leftarrow c' + c_{\mathrm{pre}}$

$s_0', s_1' \leftarrow \mathsf{SampPre}_{\hat{\mathbf{B}}}(c, \hat{\mathbf{B}})$

$s_0 \leftarrow s_0' - u_0,\ s_1 \leftarrow s_1' - u_1$

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$\hat{\mathbf{t}} \leftarrow \left(-\frac{1}{q} \cdot \hat{c} \odot \mathsf{FFT}(F), -\frac{1}{q} \cdot \hat{c} \odot \mathsf{FFT}(f)\right)$

$\mathbf{t} \leftarrow \mathsf{invFFT}(\hat{\mathbf{t}});\quad \mathbf{t} \leftarrow \mathsf{round}(\mathbf{t});\quad \hat{\mathbf{t}} \leftarrow \mathsf{FFT}(\mathbf{t})$

$\hat{\mathbf{s}} \leftarrow (\mathsf{FFT}(c), \mathsf{FFT}(0)) - \hat{\mathbf{t}} \odot \hat{\mathbf{B}}$

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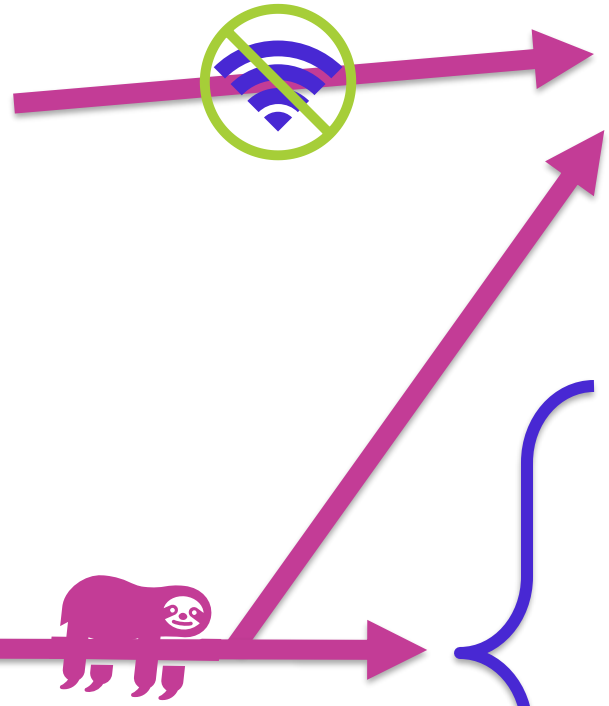

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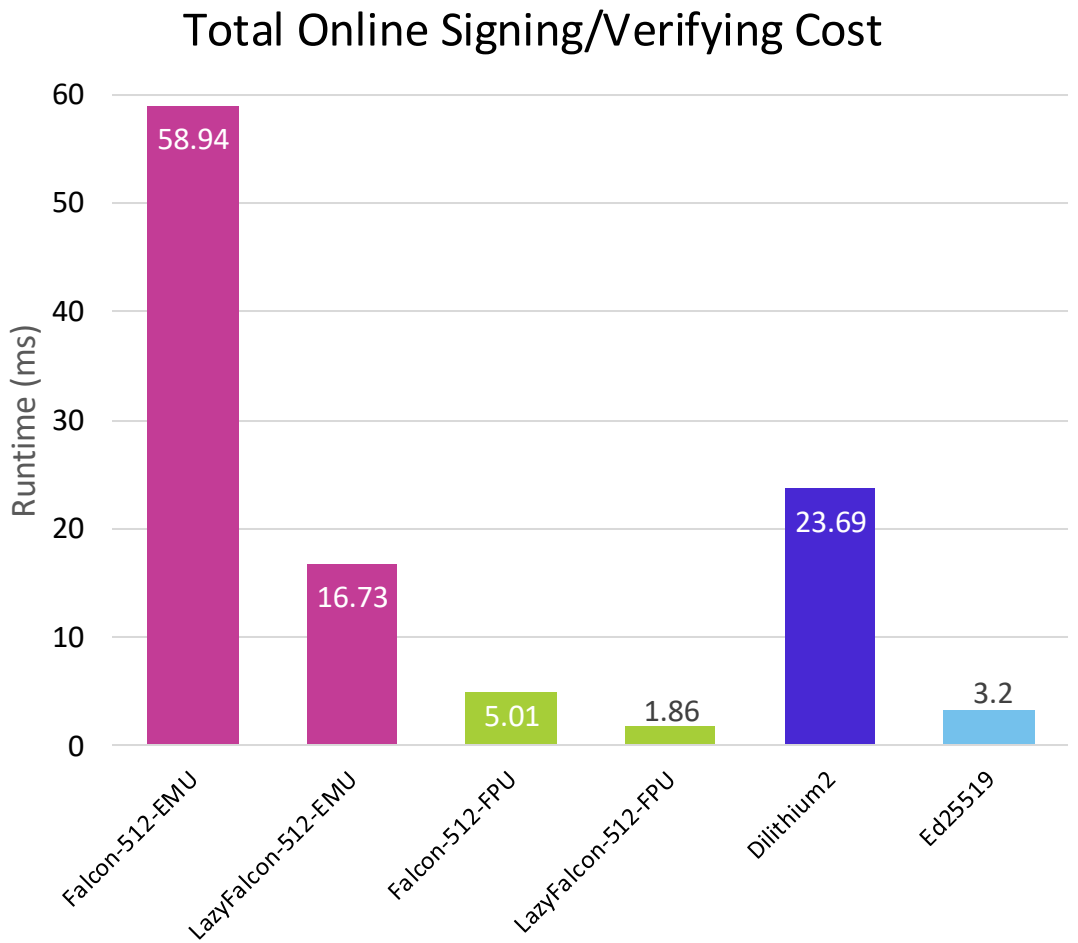
Benchmarking Post-Quantum Signatures

Many Voices.
One Community.

Benchmarking PQ signatures

- Pink is Falcon emulated
  - Most realistic for small/embedded
  - Shows a ~4x saving
- Green is Falcon using native FPU
  - Roughly the same savings vs Falcon
  - Nearly 2x faster than Ed25519
- Blue is Dilithium2
  - Much slower verify ~3 ms
  - Lazy Falcon at least 1.5x faster and nearly 13x faster with FPU



Benchmarked on Raspberry Pi 3B (ARM Cortex-A53), latest GCC

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Post-Quantum Online/Offline Signatures

Lazy Falcon “Few Times” Signatures: Pros and Cons

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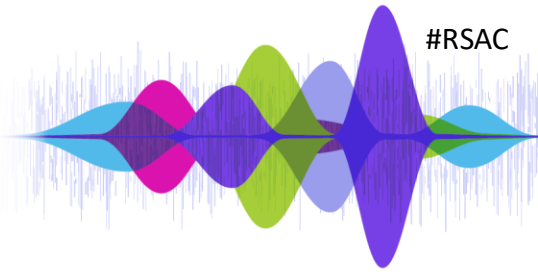

Pros and Cons of Lazy Falcon

- Pros
  - We can sign/verify faster than Ed25519
  - Lazy Falcon is compatible with Falcon
    - Same key generation, verification has one difference
  - You *kinda* get *some* side-channel protection “for free”
- Cons
  - Sig. cost is slightly bigger, but smaller than ML-DSA/Dilithium
  - “Few-times” signature is realistically at most 4 or 5 times

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Thank you

More information:

- Martin R. Albrecht, Nicolas Gama, James Howe, and Anand K. Narayanan, 2025. *Post-Quantum Online/Offline Signatures,* eprint.iacr.org/2025/117
- Reference code: https://github.com/jameshoweee/online-offline-sigs